MODELLING OF CHAMBER PRESSURE FOR ROCKET NOZZLE ALTITUDE COMPENSATION

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ABSTRACT

The optimum thrust of rocket engine is being delivered when the nozzle exit and ambient pressures are balanced. This poses a challenge with the operation of rocket engines when ascending altitudes since the ambient pressure decreases with the changing altitudes. There have been solutions designed to counter the effect of this variation in pressure with special rocket nozzles such as Aerospike nozzles, Deflection nozzles, extended nozzles, etc. However, these special nozzles add up to weight, mechanisms, rocket system engine staging and cost of production. This paper therefore looks at the rocket model that will enable thrust optimization along altitudes by determining (upstream) chamber pressure values with regards to the ambient pressure.

Index Terms - Rocket Chamber Pressure, Nozzle Altitude Compensation, Rocket Fuel Optimization

1.0 INTRODUCTION

Since the inception of the modern rocket science by Tsiolkovsky *et al*, in 1903 [3] and satellite launch systems into the outer space orbit, the pressure thrust component (the product of the nozzle exit area Ae and the pressure difference between the exit P_e and the ambient pressure P_a) among the momentum component $\dot{m}v_e$ have been a source of concern for the rocket system regarding the optimum thrust generated due to the ambient pressure exponential decrease in corresponding altitudes as represented by the basic thrust equation.

$$F = \dot{m}v_e + (P_e - P_a)A_e \qquad (1)$$

Where

- F – Thrust
- $\dot{m}$ – Mass flow rate
- v_e - Exhaust gas velocity
- P_e - Nozzle exit pressure
- P_a - Ambient pressure
- A_e - Nozzle exit area

For an optimum rocket thrust, the pressure thrust component must be zero ($P_e = P_a$) to maintain only the momentum thrust which is dependent on the mass flow rate and the exhaust velocity only [11]. It can also be deduced from the simple equation of thrust that since the ambient pressure decreases, the overall rocket thrust also increases assuming a steady exit pressure. However, the increase in the thrust of the rocket is counterproductive due to under and over expansion flow of the exhaust gases at the exit area of the nozzle. Most times in launch services the pressure thrust condition $p_e > p_a$ occurs during flight. The flow at the exit area is no more in one direction but extend to other direction in x-y plane giving rise to uncoordinated flow. As a result of this condition about 30% of the thrust generated are not useful because of bidirectional flow at the exit section which gives rise to propellant wastage and unnecessary mass considerations [2]

The pressure difference condition $p_e < p_a$ of the thrust hardly occurs except with the designers considerations where the condition $p_e = p_a$ is specified somewhere at a particular altitude, this gives rise to over expansion nozzle ratio prior to the specified altitude. Studies have shown that a little bit of over expansion of about 5% is preferable to under expansion but greater values reduce the thrust to a reasonable amount. This effect of pressure difference comes as a result of variations in altitude pressure. The pressure difference conditions consumes propellant of the rocket more than the required propulsion regarding the thrust output delivered, though the speed of the rocket increases but certainly with unnecessary cost that can be minimized [8].

Altitude compensating propulsion systems are not a new idea, with the vast majority of nozzle concepts developed half a century ago [3]. After an initial surge of interest, the dominance of multiple stage launch

systems inevitably caused a halt to the research and development of these concepts. A resurgence of interest in reducing the cost per kilogram to orbit corresponding with the emergence of single-stage-to-orbit spacecraft has resulted in a reconsideration of altitude compensating nozzles for modern propulsion systems [4]. Unfortunately, to date, there has been little testing of full-scale nozzles for altitude compensation. Furthermore, as the majority of testing conducted has been in the interests of private business and the military, information on these concepts is scarce within the public domain.
There are a number of problems that this pressure thrust differences give rise to; these include;

- **Limitation of rocket payload (Satellites, Probes, etc) to specified mass:** Since the propellant mass of rockets takes more than three quarters of rocket space for it to launch a satellite into space, this limits other subsystem's masses to enable the rocket and its propellants achieve its objective [12]
- **Increase in the number of rocket engine stages the rocket system takes on board:** Each rocket engine with it associated propellant onboard a rocket system features different nozzle expansion ratio due to the ambient pressure variations confined to its altitude of operation. These stage engines are jettisoned after their propellants are exhausted together with the nozzles; each engine stage has it stipulated operation altitudes which is based on the ambient pressure variations covering the altitudes so stipulated by the Engineers. The second reason for stage jettison is to reduce unnecessary tank carriage weight after the propellant must have been exhausted. It would have been preferably cost effective if a single propellant tank could have been jettisoned if the pressure difference component of the thrust is been taken care of [6].
- **Consumption of undeserved propellant:** Most of the propellant (fuel) used for rocket propulsion are not controlled for optimum thrust delivery due to the pressure difference component of the system. The thrust gained from pressure difference is not proportional to the amount of propellant consumed due to losses caused by bidirectional exhaust flows when under expanded nozzles are used [8].

2.0 ALTITUDECOMPENSATION NOZZLE

A number of efforts have been employed to provide solution to the thrust pressure difference of the rocket system. The National Aeronautics and Space Administration (NASA) Scientist and Engineers in 1970 developed aerospike engine that maintains its aerodynamic efficiency across a wide range of altitudes. It belongs to the class of altitude compensating nozzle engines. A vehicle with an aerospike engine uses 25–30% less fuel at low altitudes, where most missions have the greatest need for thrust. Aerospike engines have been studied for a number of years and are the baseline engines for many single-stage-to-orbit (SSTO) designs and were also a strong contender for the Space Shuttle Main Engine. However, no such engine is in commercial production, basically because if it affordability due to cost of production and weight addition on the rocket system although some large-scale aerospike are in testing phases [13].

In a similar vein, Plug nozzles also belong to a class of altitude compensating nozzles much like the aerospike which maintains its efficiency at a wide range of altitudes [14]. Plug nozzles use a shaped rocket nozzle with a poppet-shaped plug to allow the pattern of the rocket exhaust to be changed. This is used to adjust for changes in altitude; at lower altitudes the plug is pulled back to cause the exhaust to spread out, while at higher altitudes the lower air pressure will cause this to happen naturally. An alternate construction for the same basic concept is to use two nozzles, one inside the other, and adjust the distance between them. This pattern has the advantage of better control over the exhaust and simpler cooling arrangements [9]. However, these nozzles also contribute to energy demand and mass addition to the system that is critical to the mission requirement.

Expanding Nozzle was also developed and used in 1998. The nozzle employs a stationary or primary exit section that is enveloped by a movable one that slides up and down in way that balances the optimum condition of thrust performance. However, this design also consumes extra materials and mass that requires moving parts for operation. This is also a limiting solution regarding energy and cost of production. The concept can be visualized in the figure 1 below.

Gordon *et al.*, in 2001 obtained a US Patent US6591603 B2 for developing an expansion-deflection nozzle that comprises of a plug at the exit cone section which aerodynamically is compensated with the varying ambient temperature across altitudes [6]. Just like the plug nozzles, the only difference with the Gordon *et al* design is the attachment of the plug with the body of the combustion chamber or the throat section of the nozzle to reduce the thermal loads on the mechanism. However, this also has the same production issues as discussed for the solutions above.

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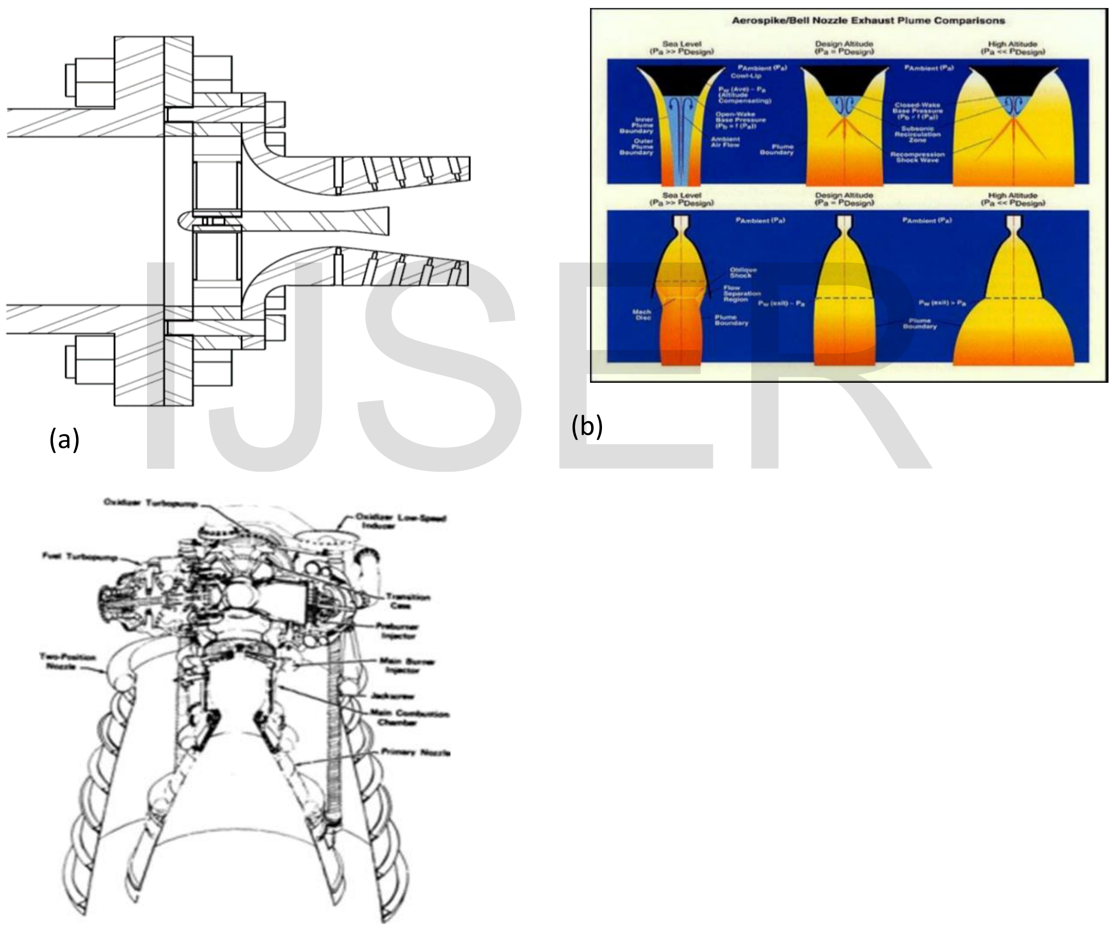
It is therefore noteworthy to mention that the above efforts and solutions for ambient pressure compensation nozzles on rocket engine proved successful. However, mechanisms, mass and thermal loading showed complexity and excess energy consumption in the above solutions. Consequently, this method proposed herewith will provide less complexity and cost effective in ambient pressure compensation design by modeling and simulation of the corresponding chamber pressures that will deliver equivalent exit pressures in view of optimizing thrust as the ambient pressure varies along the altitudes. These chamber pressure values will be used to obtain determinant mixture ratios considering a range of propellant for controlled and automated fuel intake system in view of appropriate optimization of thrust. This would further reduce the amount of fuel consumed and also increase the payload mass. Consequently, a reduction in the number of rocket engine stage used for flights to two instead of three as mostly found in rocket systems.

Furthermore, the chamber pressure and the exit pressure of a rocket engine determine the nozzle geometry in terms of the expansion area ratio and therefore affect the exit angle. This study will also look at the limits of chamber pressure variations with respect to the exit pressure by considering a range of area ratios using the computational fluid dynamic software. Below are different types of altitude compensation nozzles.



(c)

Figure 1: Different Types of Altitude compensation Nozzles; (a) Expansion Deflection Nozzles (b) a comparison of Aerospike and Bell Nozzle (c) Extended Nozzle [10]

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3.0 METHODOLOGY

The basic equations and analysis of isentropic compressible flow were considered with the assumptions that the flow is adiabatic, perfect gas behavior, one directional flow and boundary layer effects are negligible. Having obtained the existing isentropic equations for compressible flow.

3.1 MODELLING EQUATION OF ALTITUDE COMPENSATION NOZZLE

Considering compressible flow analysis of a convergent-divergent nozzle the following assumptions are made

- The flow is adiabatic and isentropic
- Uniform flow in the nozzle is in one direction
- The combustion process is chemically equilibrium
- The products of combustion behave like a perfect gas
- The specific heat is independent of pressure
- The chemical equilibrium of the gases during expansion in the chamber is unaffected

To generate maximum thrust from the propellants, convergent divergent nozzle are used as seen in figure 2 below.

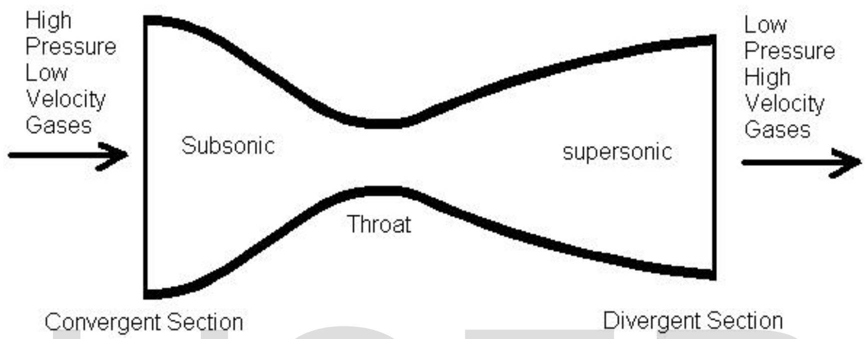

Fig. 2: A Typical convergent-divergent nozzle

Since rocket engines operate at altitudes, nozzles are always under chocked conditions. As mentioned earlier n section 1.0 for optimum performance of the rocket nozzle, the exit pressure (Pe) must be equal to the ambient pressure (Pa) Pe = Pa. Consequently, the area ration of the nozzle (exit area) A_e / (throat area) At, is also very significant. The area ratio required for a particular exit pressure and altitude or sea level is determined by the complete gas expansion equation.

$$\frac{A_e}{A_t} = \frac{A_e}{A^*} = \frac{\left(\frac{2}{k+1}\right)^{1/k-1}\left(\frac{P_c}{P_e}\right)^{1/k}}{\left\{\frac{k+1}{k-1}\left[1-\left(\frac{P_c}{P_e}\right)^{(1-k)/k}\right]\right\}^{1/2}} \qquad (2)$$

Where
A^* = Critical Area for a chocked flow at the throat of the nozzle
P_c = Chamber pressure at the combustion chamber or the stagnation pressure
k = Specific heat ratio of the propellant combustion products

*Note that the exit pressure can be a correspondence of any ambient pressure at any altitude for a complete expansion

The equation for the corresponding exit Mach number (M_e) for the exit pressure is given thus

$$M_e = \left\{\frac{2}{k-1}\left[\left(\frac{P_c}{P_e}\right)^{k-1/k} - 1\right]\right\}^{1/2} \qquad (3)$$

From equation (2) and (3), the prominent parameter in both equations is the pressure ratio $\left(\frac{P_c}{P_e}\right)$. This shows that there exist a common ratio of pressure that shares a correspondent value between the nozzle expansion ratio ($\frac{A_e}{A_t}$) and the exit Mach number of a nozzle. This common ratio of pressure is important for the development of the model since the exit pressure and the ambient pressure are the necessary parameters to determine the optimum exit gas expansion in the divergent section of the nozzle. However, any of the equations (2) or (3) can be used to obtain the mathematical model to determine the appropriate control chamber pressure to obtain a balance between the exit pressure and the ambient pressure of the rocket

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altitude. Based on analytical computation, equation (3) is thus considered for the model development. It will be noted that each nozzle has its unique expansion ratio/exit Mach number based on the designers achievable parameters such as thrust and mass flow rate. Each expansion ratio has its corresponding exit Mach number or vice versa as shown in the equation 4 below;

$$\frac{A_e}{A_t} = \left(\frac{1}{M_e}\right)\left[\frac{1+\left(\frac{k-1}{2}\right)M_e^{\,2}}{\frac{(k-1)}{2}}\right]^{\left(\frac{k+1}{2(k-1)}\right)} \qquad (4)$$

As can be seen in equation (4), k is the only value in the relationship between nozzle expansion ratio and the exit Mach number. And k varies between the range of 1to 1.6 based on the propellant properties used as fuel and it shows how well the gas can expand with lower values of the range considered.

Considering specific expansion ratio for sea level, the above equation (4) can be re-arranged as follows;

$$\frac{P_c}{P_e} = \left[\left(\frac{k-1}{2}\right)M_e^{\,2} + 1\right]^{k/k-1} \qquad (5)$$

The specific heat ratio, k is a constant and the two variables in the modified equation (5) are the pressure ratio and the exit Mach number, M_e. The nozzle pressure ratio is therefore analyzed numerically with respect to the exit Mach number as shown in figure 3 in the result section of this paper.

Further modification of equation 5, relates about the balance of exit pressure and the ambient pressure along altitudes for optimum thrust performance.

For optimum gas expansion: $P_e = P_a$

Furthermore, it has been established that the ambient pressure along altitude is thus [5];

$$P_a = P_o e^{-h/h_o} \qquad (6)$$

Where,
P_a = Ambient pressure
P_0 = Surface Pressure or reference pressure
h = Altitude height
h_0 = scale height of the atmosphere (7km)

Therefore, equation (5) is further modeled as shown in equation (7);

$$P_c = \left[\left(\frac{k-1}{2}\right)M_e^{\,2} + 1\right]^{k/k-1} P_o e^{-h/h_o} \qquad (7)$$

In equation (7), the altitude h is the dependent variable when the exit Mach numbers, specific heat ratio, reference pressure and heights have been known. The combustion chamber pressure can be determined according to the altitude h attained by the rocket. This chamber pressure can be regulated to obtain an optimum combustion gas expansion at the nozzle exit based on the altitude of the rocket and perhaps the ambient conditions of the altitude. This is the model developed to obtain the optimum thrust generated by the nozzle during rocket flight.

For the purpose of numerical studies of the preceding equations. MATLAB software was used for the numerical evaluation of the chamber pressure according to the varying ambient pressure. However, the discretization of the altitude component is made in an increment of 1 kilometer due to the fast changing velocity of the rocket which is an average of about 7.8km/s orbital velocity to reach space that begins at a range of 100km altitude and also to reduce the time of computation. The results of the computation are generated in MATLAB in terms of the Chamber Pressure (Pc), Specific Heat Ratio (k), Mach number (Me), Pressure ratio (which is a ratio of Chamber Pressure and Ambient Pressure PR), and Altitude in kilometers (h). The data is therefore presented in an excel or text file data sheet but cannot be contained in this paper due to the limitation of word publication.

4.0 RESULTS AND DISCUSSIONS

Figure 3 is a graph generated from equation (2) that shows the relationship between the nozzles pressure ratio and the area ratio. Equation (2) is an existing equation that the nozzle area ratio increases exponentially as the pressure ratio increases. However, their rate of variations depend on the type of propellant being used in the combustion chamber which determines the value of the heat capacity k. In this analysis, the heat capacity is considered as 1.2 which is typical for LOX/RP-1. For a nozzle area ratio of one, the corresponding pressure ratio is 1.78 which signifies that the throat section of the nozzle which is the intersecting section between the convergent and the divergent part of the nozzle bears the least pressure ratio. The convergent part of the nozzle is of less consideration in this paper since the altitude compensation bothers on the extent of the divergent of the exit area. It will also be observed that the more the exit area (Ae) increases for a constant throat area, the less the exit pressure (Pe) assuming the chamber pressure is constant. It is also significant to start the modelling for the adaption of nozzle exit pressure with the ambient pressure from equation (2) since the exit area is fundamental to the exit pressure of the nozzle. Furthermore, other parameters as relate to pressure ratio such as the exit Mach number is also analyzed as shown in figure 4.

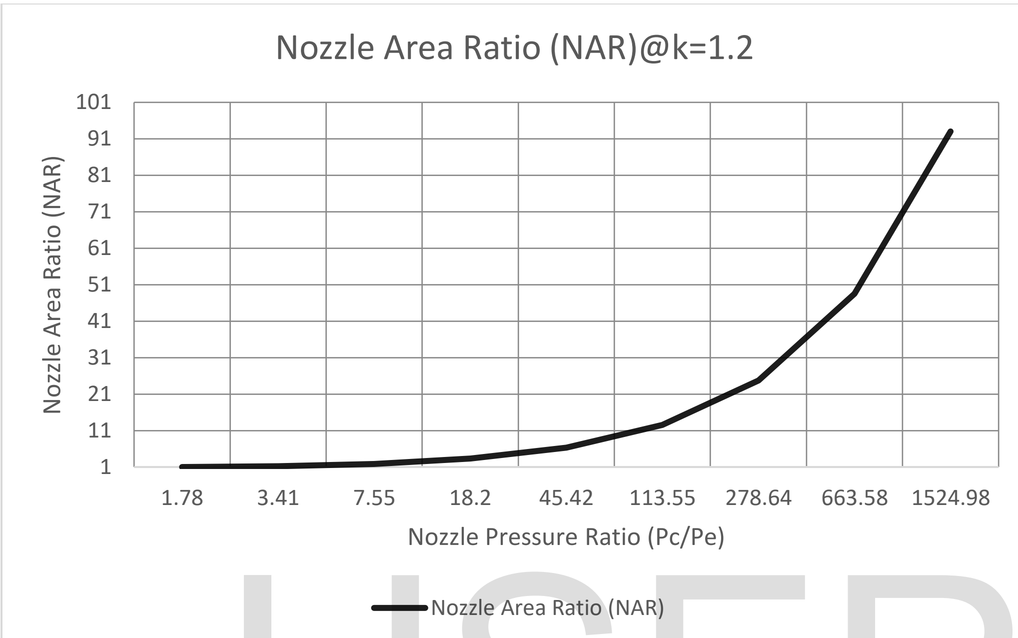

Figure 3: Graph of variation between Nozzle Area Ratio (Expansion Ratio) and Pressure Ratio @ k = 1.2

Figure 4 shows a correlation of the pressure ratio and the exit Mach number as generated by equation (3) which is also existing in literatures. The typical values of chamber pressure increases as the operational Mach number of the rocket increases. The chamber pressure which is part of the pressure ratio (Pc/Pe) of a rocket engine that determines the magnitude of the exit velocity V_e which is independent of the size of the engine, be it micro thruster on a satellite or a launch vehicle. One factor that determines the size of the thruster or the nozzle is the mass flow rate which is as a result of the throat area [1]. Since the rocket operates supersonically, its operates from the Mach number of more than one which bothers on the divergent section of the nozzle as earlier mentioned previously. This analysis also shows that for a constant chamber pressure, the operating exit Mach number of a nozzle increases as the exit pressure decreases. This means that the longer the divergent part of the nozzle the less the exit pressure and the higher the exit Mach number. It can be shown further the correlation between the expansion ratio and the exit Mach number of the nozzle as shown in equation (4). This will easily enable the determination of the corresponding area ratio or exit Mach number when dealing with the nozzle pressure ratio.

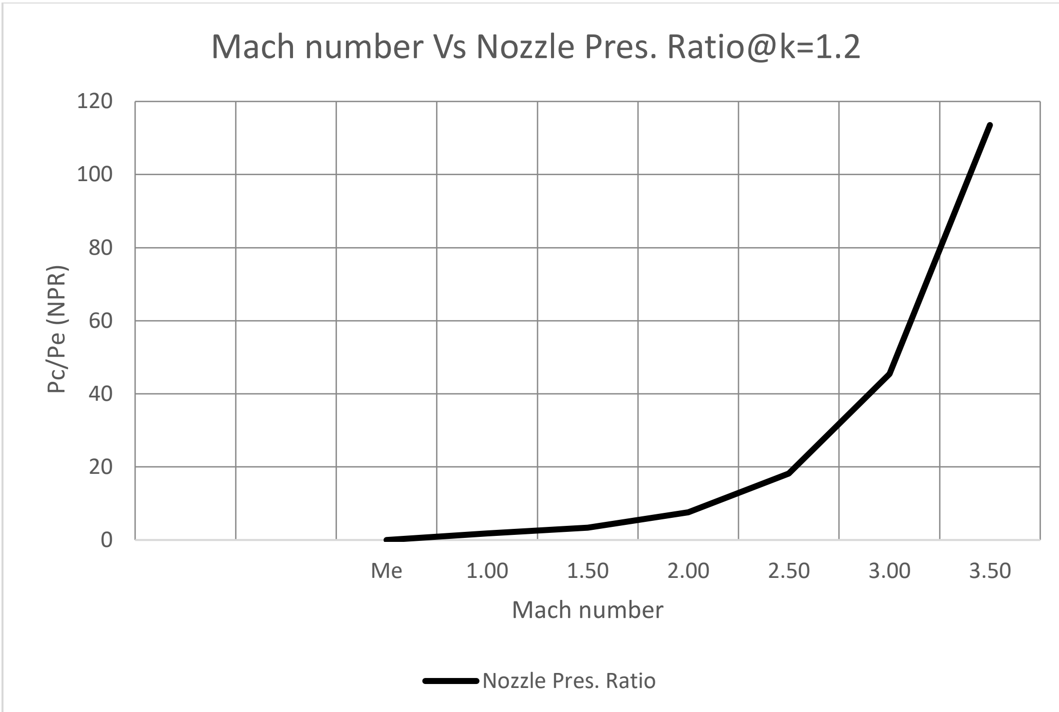

Figure 4: Graph of variation between Nozzle Pressure Ratio and Exit Mach Number @ k = 1.2

Figure 5 is the graph showing the variation between exit Mach number and the expansion ratio or the area ratio. The variations are considered for Mach numbers between one to five due to the supersonic scope of the analysis. It's also confirms that the throat section of the nozzle where the exit Mach number is one corresponds to the nozzle area ratio of one. This shows significantly the agreement between the previous pressure ratio analyses as it relates the exit Mach number and the area ratios of the nozzle. Having laid the bases of the numerical analysis for chamber pressure variation for the adaption of nozzle exit pressure due to altitude variation during rocket flight, equation (7) is the modelled equation that estimates the chamber pressure at corresponding altitudes for the adaption of nozzle exit pressure with the altitude's ambient pressure. This model is dependent of the thrust of the considering rocket engine and as such the determining rocket chamber pressure depends thrust and the desired adaptive nozzle altitude. Figure 6 is a graph of variation of altitude and the estimated chamber pressures of the rocket engine.

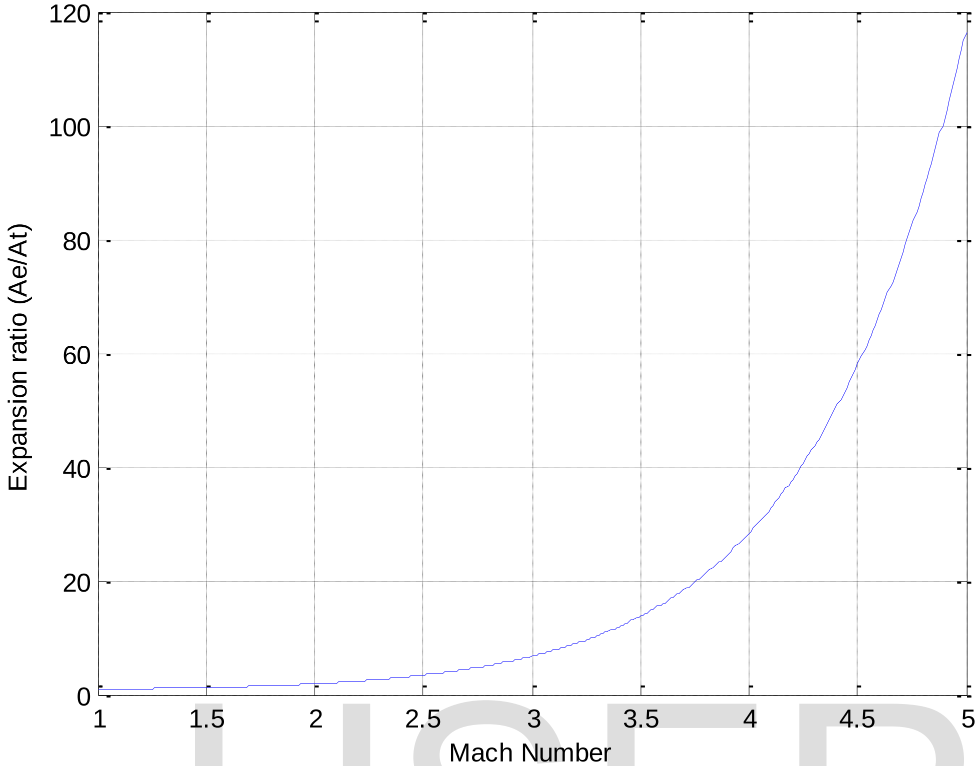

Figure 5: Graph of variation between Nozzle Exit Mach Number and Area (Expansion) Ratio @ k = 1.2

it will be observed that the typical pressure values of the chamber pressure increases as the operating Mach number also increases as seen in numerical representation ranging from Mach numbers 2.5, 3.0 and 3.5 respectively. Their corresponding increases are 1.6Mpa, 2.9Mpa and 9.3Mpa this means that the exit or exhaust velocity which is the product of Mach number and the speed of sound of a rocket greatly depends on the chamber pressure. This eventually increases the thrust and the performance of the rocket using good properties of propellant such as low molecular weight and its heat ratio k as shown by the ideal thrust. Furthermore, figure 6 shows Mach numbers and the specific heat ratios are held at a constant value from 2.5 to 3.5 and 1.24 respectively. It is observed that there is an exponential decrease in the typical values of chamber pressure from the first 23-30km change in altitude as corresponds to its respective ambient pressures. For example the chamber pressure decreased from 3.8 Mpa to 1.06 Mpa for Mach number 3. This shows that for an optimum operation condition the thrust of the rocket falls within a threshold of maintaining its mass flow rate to sustain the required thrust within the first 23-30 km altitude as observed in the numerical representation for different operating Mach numbers.

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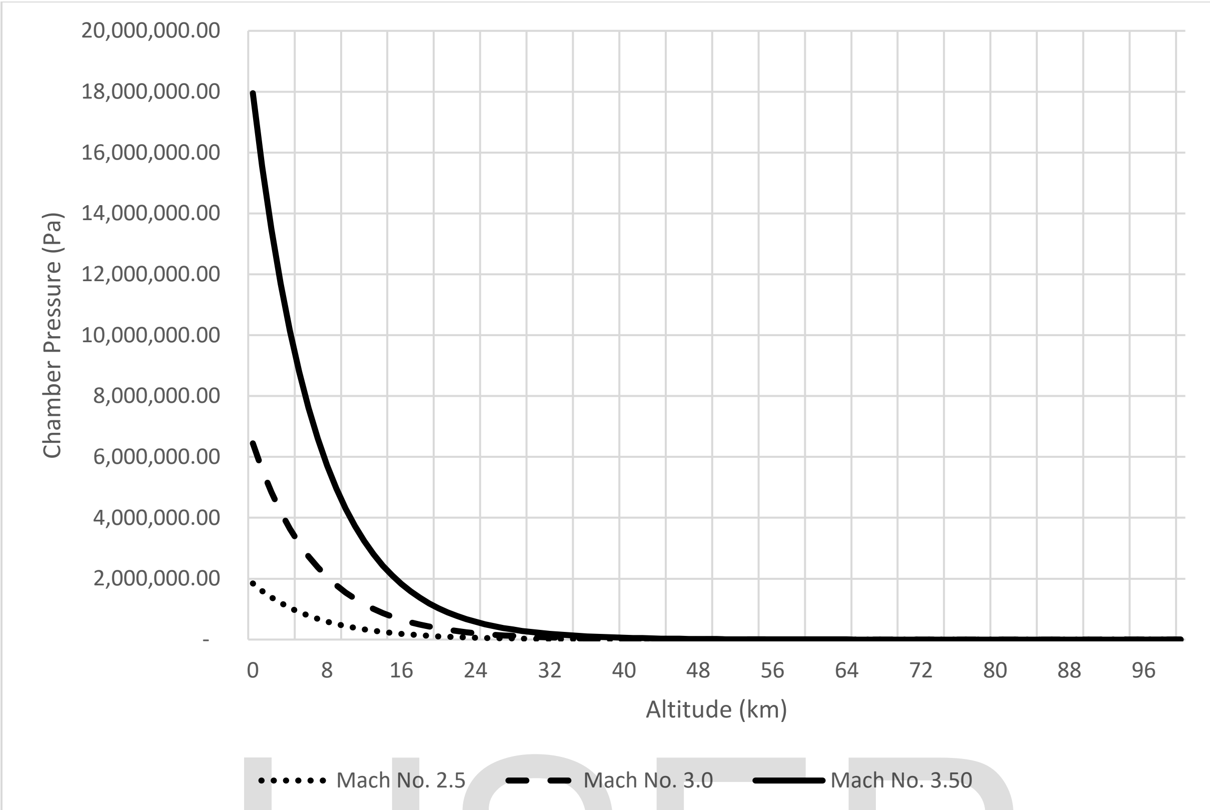

Figure 6: Graph of Variation of Estimated Chamber Pressure Vs Altitude at different Mach numbers.

This numerical model analysis specifically indicate how typical chamber pressure values can be controlled upstream during the combustion process by varying the combustion chamber pressure. The model is specifically analyzed using liquid oxygen and RP-1 (Kerosene) as propellant due to its common propellant usage, stable at room temperature, far less of an explosion hazard and denser: RP-1 is significantly more powerful than liquid Hydrogen (LH_2) by volume..

The pressure thrust component of the Thrust equation gives additional thrust when Pe >Pa but the reverse is the case when Pe < Pa and optimum when both are balanced. Figure 6 also shows that at each designed Mach number and specific heat ratio, there is a corresponding pressure ratio for all altitudes which has a maximum limit of about 24-30km which lies the effective operating thrust threshold due to variation in chamber pressure. The chamber Pressure values can differ base on thrust required due to the mass flow rate properties and exit velocity of gases. However, the chamber pressure values obtained in the model in equation (7) are based on pressure component of the thrust. It does not consider the mass flow rate of the propellant. The effect of energy conservation comes into play from the decrease in chamber pressure which give rise to a reduction in mass flow rate of the combusted gases. However, a reduction in mass flow rate is cautious beyond the allowable limit of 23-30km so as to avoid subsonic flow and shock waves in the divergent section of the nozzle. Since the mass of the combusted gases are reduced, there is a potential for minimization of the mass of the propellant used for a rocket engine.

5.0 CONCLUSION

The equation models and numerical analysis presented above, will give a guide and a baseline approach in optimization of energy utilized in rocket engine and provides deterministic values considering chocked flow at the nozzle throat during the flight operations along the altitudes. The model shows an exponential decrease in chamber pressure values and a significant drop in chamber pressure as the ambient pressure gets thinner at altitude of about 30km within the threshold of minimum thrust generated by the rocket. However, it will be recommended that rocket engines operated in space should be customized to the space environment due to the peculiar environment in space. This model will enable the elimination of the following challenges in rocket design;

- Reduction of propellant consumption and enhance optimum performance
- Reduce the number of engine stages to at least two
- Reduce the mass added by engine stages and additional driven mechanism for altitudes compensation nozzles

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